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The Orbital Angular Momentum Modes of the Bessel Vortex Beam are Modulated by using the Metasurface

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Abstract. The orbital angular momentum associated with the helical phase wavefront of electromagnetic waves has orthogonal infinite spatial modes in theory, and the channel capacity is greatly increased by spatial mode multiplexing. Among the different methods for generating and manipulating the orbital angular momentum state of light, using a metasurface to produce more modes coupling between spin and orbital angular momentum allows faster manipulation of the orbital angular momentum state, which is simpler and faster than manipulating conventional orbital angular momentum generators. In this work, the orbital angular momentum carried by the Bessel vortex beam changes from $+3$ to -1 by using the metasurface, while $+1$remains $+1$. It is found that the interaction between the metasurface and the vortex beam is not only related to the specific geometry of the metasurface, but also to the OAM state of the beam. The numerical results agree well with the theoretical results. This paper provides a theoretical basis for studying the orbital angular momentum carried by light beams in optical communication and information processing.

Keywords: modulate, orbital angular momentum, Bessel vortex beam, metasurface.

1. Introduction

The angular momentum (AM) of electromagnetic wave can be decomposed into spin angular momentum (SAM) and orbital angular momentum (OAM)[1]. The SAM is related to the polarization properties of light[2]. The OAM is related to the wave-front shape of the electromagnetic wave, when the spatial phase is $exp(il\varphi)$, where l is topological charge, also known as OAM mode, and φ is the spatial azimuth angle[3]. For example, each photon of the first-order Bessel vortex beam carries an orbital angular momentum of $1\hbar$, where $\hbar$ represents the reduced Planck constant[4-5].

The research shown that the interaction of electromagnetic waves with inhomogeneous or anisotropic media resulting in spin-orbital angular momentum conversion[6-7]. Metasurface is a two-dimensional material with spatial anisotropy, which converts SAM beams into OAM beams and generates central singularities by adjusting the wavefront phase and amplitude of electromagnetic waves[8-9]. An electromagnetic metasurface is designed and fabricated, and experimentally demonstrated to generate multiple orbital angular momentum vortex beam in radio frequency domain[10]. A metasurface consisting of an annular subwavelength grating was proposed, successfully producing an OAM-carrying beam with an $l = \pm 2$ topological charge, achieving an efficiency of up to 90%[11]. An approach to generate and transform surface wave to orbital angular momentum (OAM) vortex wave using flat dispersive metasurface was proposed, generating multiple OAM modes of $l = 0$, 1 and 2, respectively[12]. A reflective metasurface is proposed, which is capable of generating an $l = -1$ or $l = +1$ OAM mode reconfigurable beam, and an $l = +1$ spinning OAM beam[13]. As far as we know, plane waves designed by the metasurface can be transformed into vortex beams carrying OAM, but the interaction between vortex beams and the metasurface is rarely reported. In this paper, the modulation of the orbital angular momentum of the Bessel vortex beam by a metasurface is studied, revealing the mystery of the interaction between the spin and orbital angular momentum of the beam by the metasurface.

The paper is organized as follows. In Section 2 , the interaction between the metasurface and the spin and orbital angular momentum of vortex electromagnetic waves is studied theoretically by using Jones matrix. Then, in Section 3 the effects of the metasurface on the electromagnetic intensity, phase, spin and orbital angular momentum of the first and third order Bessel vortices are analyzed by numerical calculation. Finally, conclusions are drawn in Section 4.

2. TheoreticalAnalysis

Assuming that the incident beam is the bessel vortex beam, its cross-section electric field is expressed as

$$E_n(\gamma,\varphi,z) = A_0 exp(ik_z z) J_n(k_r r) exp(in\varphi) \tag{1}$$

where A_0 is a constant, J_n is the nth-order Bessel function, and $n$is a non-zero integer number, k_z and k_r are the longitudinal and radial components of the free-space wave vector, such that$k = \frac{2\pi n}{\lambda} = \sqrt{k_z^2 + k_r^2}$.

As shown in Fig.1, assuming that electromagnetic waves propagate along the Z-axis, the metasurface is placed on the propagation path of the Bessel vortex beam.

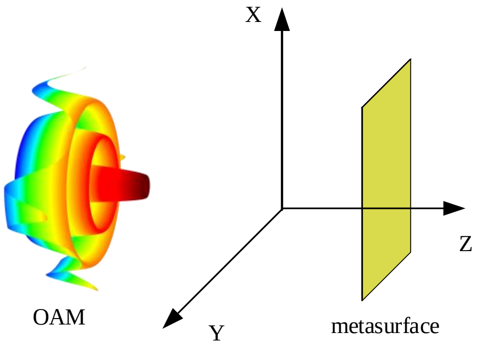

Fig. 1: Schematic diagram of the propagation system.

A schematic diagram of the metasurface element is shown in Fig.2. The unit as a whole is circular, consisting of six identical small components with a distance of 0.1um rotating around, and a radius of 1.5um. Fig.2(a) Schematic diagram of metasurface element structure Fig.2(b) Side view of metasurface element and dielectric substrate. It is reasonable to ignore the diffraction effect when studying the interaction between the bessel vortex beam and a metasurface. This is because the metasurface is a two-dimensional material with almost zero thickness, while the Bessel vortex beam has a relatively large transverse size [14]. In addition, it is well known that the Bessel beam is typically non-diffracting and the diffraction effect in the central area of the Bessel beams is not essential.

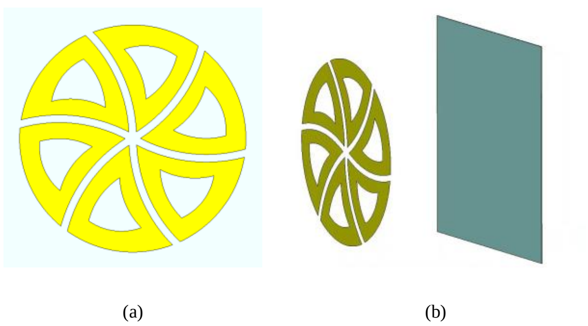

Fig. 2: The metasurface element and dielectric substrate.

Considering that each microstructural unit of the Metasurface has the same phase delay π, and ignoring the diffraction effect of the Bessel vortex beam when transmitting in the Metasurface, the Jones matrix can be expressed as

$$M_{MS} = \begin{pmatrix} \cos(2q\varphi) & sin(2q\varphi) \\ \sin(2q\varphi) & -\cos(2q\varphi) \end{pmatrix} \tag{2}$$

where θ is the angle between the fast axis of metasurface and the fixed axis x. In this paper, the metasurface with phase delay equal to π is used [15]. In particular, the metasurface pattern is given by the following expression:

$$\theta = q\varphi + \varphi_0 \tag{3}$$

Where q is the topological charge of metasurface ,$\varphi = arctan(y/x)$, $\varphi_0$indicates the initial orientation on the fixed axis x. Since the light beam hits the metasurface with normal incidence, making the metasurface parallel to the XY coordinate plane, so that $\varphi_0 = 0$.

The bessel vortex beam with OAM $l\hbar$ (Where l is topological charge, which is also the order of the bessel vortex beam,$-$ represents negative order, spiral phase rotates counterclockwise, $+$ represents positive order, spiral phase rotates clockwise) is used as the incident beam to irradiate the metasurface. Thus, the Jones vector of the incident wave can be written as follows:

$$E_{in}(\gamma, \varphi) = E_0(r, \varphi) \times [1, li] \tag{4}$$

At the metasurface output, obtain field as

$$E_{MS} = E_0 e^{\pm 2q\varphi} \begin{bmatrix} 1 \\ \mp i \end{bmatrix} \tag{5}$$

It is seen from Eq. (5) that the output wave is still vortex beam, the OAM changes $\pm 2q$,but the rotation direction of the helical phase is opposite to the direction of the incident wave. This is all because the retardation of metasurface is π. Moreover, the helical phase of the output wave is topological charge $\pm 2q$. It is notable that the $\pm$ sign of topological charge is determined by the direction of rotation of the helical phase of the incident wave. From the perspective of AM, each input photon carries OAM $l\hbar$, while the carried SAM is zero. The total AM is equal to the OAM $l\hbar$. At the output of metasurface, the output wave has acquired a helical phase $\pm 2q$, followed by a reversal in the direction of the helical phase rotation. Therefore, each output photon carries OAM $(\pm 2q - 1)\hbar$ .The total AM is then equal to $(\pm 2q - 1)\hbar$ and the variation of AM is $2(\mp q - 1)\hbar$.It is well known that AM must be conserved, therefore, the variation of AM must be exchanged with the metasurface. That the spin–orbital interaction occurs in the inhomogeneity is the intrinsic physical cause of the conversion.

In fact, the action of the metasurface adjusts the orbital angular momentum OAM beam to $\pm 2q$, and the metasurface realizes the polarity reversal of OAM. In the whole process, the AM (including SAM and OAM) changes when light pass through the metasurface with π phase retardation. It must be pointed out, however, that a light exchange AM with the metasurface[14].

3. Results

Fig.3 shows the intensity distribution of the third-order Bessel vortex beam and the first-order Bessel vortex beam before and after passing through the metasurface. It can be seen that the intensity distribution of the first-order Bessel vortex beam is still the first-order Bessel vortex beam after passing through the metasurface. The intensity distribution of the third-order Bessel vortex beam is similar to that of the -1 order Bessel vortex beam after passing through the metasurface. Moreover, the outgoing beam has a circular shape and the intensity is zero at the origin. This null intensity is caused by the phase singularity at the center of the beam and is characteristic of the beam before the helical phase. The first-order Bessel vortex beam does not appear to change after passing through the metasurface. In fact, when the vortex beam carrying orbital angular momentum is introduced into the metasurface system, the orbital angular momentum of the vortex beam first changes ± 2 (the topological charge of the vortex changes ± 2), and then the polarity reverses. That is, the orbital angular momentum of the first-order Bessel vortex beam changes from $+1$ to -1 and then reverses polarity to $+1$.

Fig.4 show that the phase distribution of first-order and third-order Bessel vortex beams before and after passing through the metasurface. Fig.4 (a) to (b) are the phase distribution of the first-order Bessel vortex beam, which seems to remain unchanged. In fact, its topological charge changes from $+1$ to -1， and then to $+1$ again. The size of the OAM value l decreases by 2, and the sign reverses. However, in Fig. 4 (c) to (d), the topological charge of the third-order Bessel vortex beam changes from $+3$ to $+1$, and then reverses to -1 , Therefore, the twisting direction and the number of spirals in the metasurface reversal pattern change, which is twice the topological charge of the metasurface. Fig. 4(b) and (d) show the polarization mode of the converted beam output of the metasurface. These confirm that the output beam has an OAM value of twice the metasurface topological charge, $l = \pm 2q$, as expected from our previous discussion.

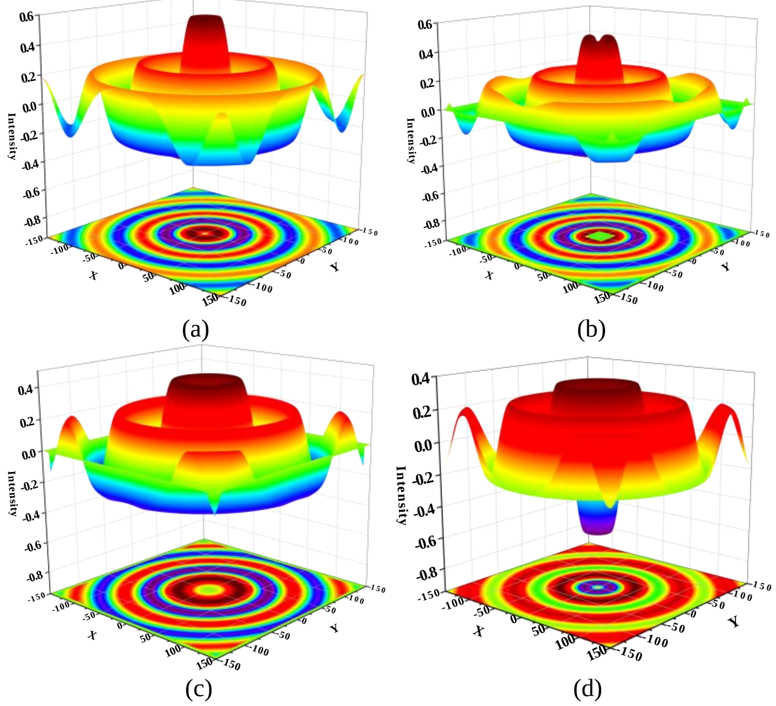

Fig. 3:The intensity distribution of first-order Bessel vortex beam and third-order Bessel vortex beam before and after passing through the metasurface. (a)(b) the first-order Bessel vortex beam. (c)(d) the third-order Bessel vortex beam.

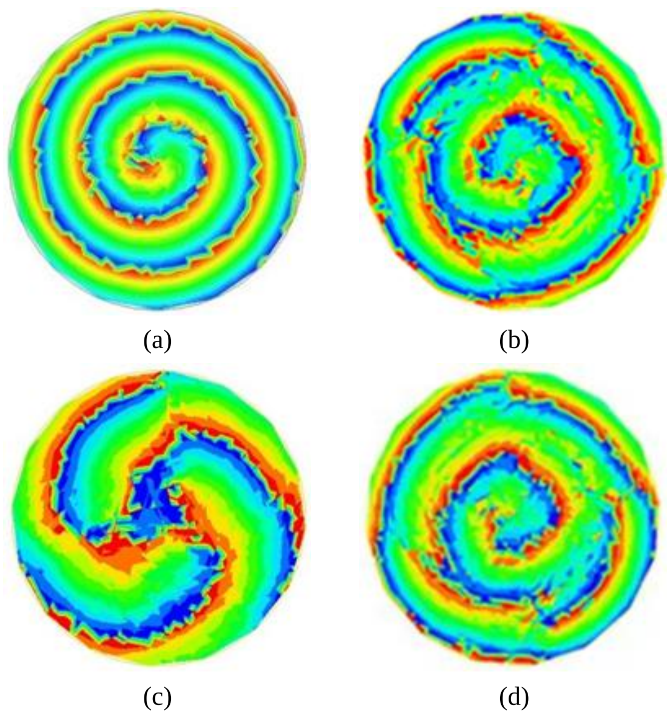

Fig. 4: Phase distribution of the first-order and third-order Bessel vortex beams before and after passing through the metasurface. (a)(b) First-order Bessel vortex beam. (c)(d) Third-order Bessel vortex beam.

Obviously, the orbital angular momentum of the vortex beam is coupled to the inhomogeneous and anisotropic metasurface. But surprisingly, they are not like electromagnetic waves that are converted into vortex beams through a metasurface. In general, the condition of electromagnetic wavepassing through the metasurface to produce helical phase depends only on the geometry of the metasurface. However, when the vortex beam carrying orbital angular momentum interacts with the metasurface, whether the angular momentum transfer occurs depends not only on the structure of the metasurface, but also on the topological charge of the vortex beam. For some vortex beams carrying orbital angular momentum, the total angular momentum transferred to matter is completely lost in certain geometry.

In order to further reveal the interaction between the vortex beam carrying orbital angular momentum and the metasurface, the Fourier transform is used to decompose the individual OAM mode, so as to quantitatively analyze the purity of OAM mode. The corresponding equations are given as follows

$$A_n = \frac{1}{2\pi}\int_0^{2\pi} \psi(\varphi) e^{-jn\varphi} d\varphi \tag{6}$$

$$\psi(\varphi) = \sum_n A_n e^{jn\varphi} \tag{7}$$

Where $\psi(\varphi)$ is the sampling electric field function on the cross section perpendicular to the propagation axis. Here, the OAM modes from $n = -5$ to $n = 5$ are considered and the energy weight (EW) of the OAM mode n is defined as follows.

$$EW = \frac{A_n}{\sum_{n'=-5}^{5} A_{n'}} \tag{8}$$

Fig. 5 shows the OAM mode distribution after the interaction between the Bessel vortex beam in OAM mode $n = +1$ and the metasurface by Fourier transform analysis. As can be seen from the picture, the expected OAM mode $n = +1$ is the main part and other OAM modes are small enough to be ignored except for some possible interference OAM modes $n = +1 \pm l(l = 1,2,3)$ which are introduced by the directional anisotropy of the metasurface.

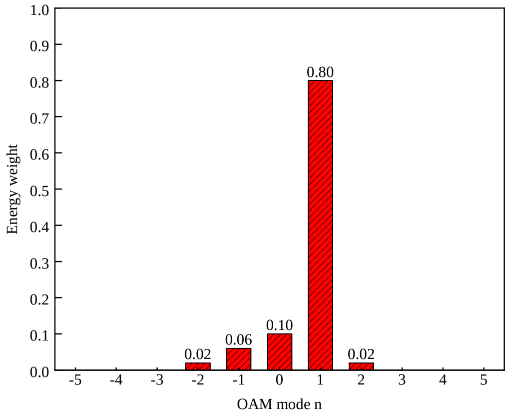

Fig. 5: spectral analyses of the Bessel vortex beam with the OAM mode $n = +1$.

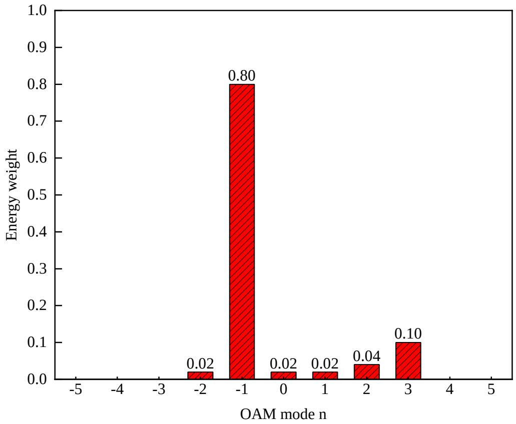

Fig. 6: spectral analyses of the Bessel vortex beam with the OAM mode $n = +3$.

Fig.6 shows the OAM mode distribution after the interaction between the Bessel vortex beam in OAM mode $n = +3$ and the metasurface by Fourier transform analysis. As can be seen from the picture, the expected OAM mode $n = -1$ is the main part and other OAM modes are small enough to be ignored except for some possible interference OAM modes $n = -1 \pm l(l = 1,2,3,4)$ which are introduced by the directional anisotropy of the metasurface. As shown in Fig.6, since a higher conversion efficiency can be obtained, the generated $n = -1$ bessel vortex beams possess a higher energy weight (around 80%).

4. Conclusion

In this paper, the intensity and phase distribution of a Bessel vortex beam modulated by a metasurface are studied. The third-order Bessel vortex beam with orbital angular momentum of $+3$ carries a topological charge of -1 in its spatial phase wavefront after being adjusted by the metasurface, and its intensity distribution is the same as that of the negative first-order Bessel vortex beam. However, the first-order bessel vortex beam carrying orbital angular momentum $+1$ is still the first-order bessel vortex beam after the metasurface adjustment, because the orbital angular momentum of the first-order Bessel vortex beam becomes -1 after passing through the metasurface, and then the polarity reverses. The results show that when a vortex beam carrying orbital angular momentum interacts with the metasurface, whether angular momentum transfer occurs depends not only on the structure of the metasurface, but also on the topological charge of the vortex beam. Since the conservation of angular momentum, the orbital angular momentum carried by the vortex beam changes and must be exchanged with the metasurface. Therefore, the OAM mode of the Bessel vortex beam can be modulated by the metasurface. For the Bessel vortex beam incident on the metasurface, it is special significance to study the change of the OAM. These results are valuable for the applications of optical communication and information processing based on optical OAM.

5. Acknowledgements

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